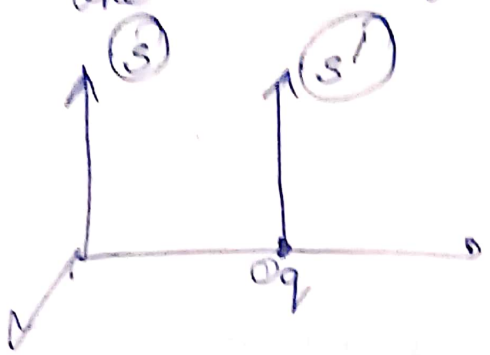
The Field of a Uniformly moving point charge :-



Let us consider a particle of charge q moving with uniform velocity u in the frame S.
We would like to find the electric and magnetic field in the field frame of reference S due to the moving charge q. Let us consider an inertial frame S' attached to the charge q so that the charge q is at rest in the origin of the frame S'. So the field is static electric field of a point charge.

∴ In the present scenario the charge q is at rest at the origin of S' and moving relative to S with a velocity u along the common x-x' axis.

∴ The position of the charge along the x axis at time t be given by $x = ut$.

∴ w.r.to S frame the space co-ordinates of q are $(x, 0, 0)$

In the S' frame the field is purely electric ie

$$E' = \frac{q}{4\pi\epsilon_0} \frac{r'}{r'^3} \qquad B' = 0 \quad \text{---} \rightarrow (1)$$

The electric field lines diverging from q with spherical symmetry.

Hence $r' = (x'^2 + y'^2 + z'^2)^{1/2}$ ---→ (11)

and r is the distance from O' to the point at which the field strength is measured.

In order to obtain the fields E and B in inertial frame S, corresponding to the moving charge, we use the following transformation rule

$$\left.\begin{aligned} E_x &= E_x' \\ E_y &= \gamma(E_y' + vB_z') \\ E_z &= \gamma(E_z' - vB_y') \end{aligned}\right\} (111)$$

and

$$\left.\begin{aligned} B_x &= \gamma B_x' \\ B_y &= \gamma\left(B_y' - \frac{v}{c^2}E_z'\right) \\ B_z &= \gamma\left(B_z' + \frac{v}{c^2}E_y'\right) \end{aligned}\right\} (1V)$$

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As r' has components x', y', z' and that the field in S must be expressed in terms of x, y, z. Hence using Lorentz transformation we have

$$r' = (x'^2 + y'^2 + z')^{1/2} = \left[\gamma^2(x-vt)^2 + y^2 + z^2\right]^{1/2} \quad \text{(V)}$$

Now the x', y' and z' components of electric fields are

$$E_x' = \frac{qx'}{4\pi\epsilon_0 r'^3}, \quad E_y' = \frac{qy'}{4\pi\epsilon_0 r'^3} \quad \text{and} \quad E_z' = \frac{qz'}{4\pi\epsilon_0 r'^3} \quad \text{(VI)}$$

Now according to equation (VI)

$$E_x = E_x' = \frac{q\gamma(x-vt)}{4\pi\epsilon_0\left[\gamma^2(x-vt)^2 + y^2 + z^2\right]^{3/2}}$$

$$E_y = \gamma E_y' = \frac{\gamma q y}{4\pi\epsilon_0\left[\gamma^2(x-vt)^2 + y^2 + z^2\right]^{3/2}}$$

$$E_z = \gamma E_z' = \frac{\gamma q z}{4\pi\epsilon_0\left[\gamma^2(x-vt)^2 + y^2 + z^2\right]^{3/2}} \quad \text{(VII)}$$

The equation (VII) gives the components of electric field E w.r. to the frame S.

The components of the magnetic field B in S follows can be achieved by substituting E' and B' in the transformation relation for the magnetic field components as stream given by equation (IV).

~~$B_{11} = B_1, B_x = 0$~~

$$\begin{cases} B_x = B_x' = 0 \\ B_y = -\frac{u}{c^2} E_z = \\ B_z = \frac{u}{c^2} E_y = \end{cases}$$

[using the values of B_y and E_z from equation (VII)]

The equation (VIII) and equation (VIII) are the exact expression for the electric and magnetic field of a uniformly moving charge. It is also clear that the fields depend upon the time t and the x, y and z components.